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A model for color screening in a QCD plasma: the roles of thermal gluons and of confinement

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Abstract. The study of the screening in the $q\bar{q}$ plasma, in a model which takes into account only static interactions, is continued with the introduction of two new dynamical elements: the presence of thermal gluons and a phenomenological description of the confinement. In the first case the qq correlation and the $q\bar{q}$ correlation are similar to each other and also similar to the correlation in the absence of gluons: the decay with the distance deviates slightly from a standard exponential decay. In the second case the two-body confining potential gives rise to correlation functions oscillating with the distance, so that only the total correlation, i.e. the space integral, has a more transparent interpretation; moreover, the qq correlations and the $q\bar{q}$ correlations show very definite differences.

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1 Introduction

An analysis of the screening in $q\bar{q}$ plasma has been presented in [1], where a model developed in analogy with the classical treatment of an electric plasma was used [2]. Within the restricted dynamics where we have only quarks feeling a static interaction coming from single-gluon exchange, the two-body correlation was found to decay with a law slightly different from the standard exponential decay.

The previous model is widened by proposing two improvements. In a first instance, we introduce thermal gluons among the components of the plasma; so, further color configurations have to be evaluated. Since we are interested in the comparatively long-distance effects, the relevant dynamical feature is, as in [1], the exchange of a virtual gluon in the t-channel. In the second instance, starting from the evidence that the confinement is not yet explained in terms of exchange of one, two, ... virtual gluons, we make use of a phenomenological confining potential.

The paper contains three main sections. Section 2 reviews the general formalism which was used in [1] to calculate the correlation functions. An extended summary is presented together with some small modifications needed to make it suitable for the next developments. The dependence of the total quark and gluon densities on the temperature is briefly reviewed: taking a definite quark population as the initial condition, the gluon population is considered to be of thermal origin and is given in terms of a Bose–Einstein density, i.e. the gluon population from which we start does not include the effect of gluon decays or coalescence implied by the vertices $g \to gg$ and $g \to ggg$. The presence of a $q\bar{q}$ pair is described by a corresponding Fermi–Dirac density. So, we start from a free quark and a free gluon population and in the subsequent sections we estimate some effects of their interactions. Section 3 embodies the extension of the formulation already used for the qq and $q\bar{q}$ systems to the qg and $\bar{q}g$ systems. One of the conclusions is that the introduction of qg and $\bar{q}g$ does not change the qualitative behavior of the qq and $q\bar{q}$ systems: the decay law of the correlations, as a function of the mutual distance, is not significantly different from an exponential curve. Hence, the gg interaction mediated by one-gluon exchange is not expected to give rise to a significant modification in the long-distance behavior of the qq and $q\bar{q}$ correlations. Its treatment would be conceptually easy but technically very heavy and will be omitted. Interactions originated by the four-gluon vertex will not give rise to long-distance effects. Section 4 is devoted to the study of the possible effects of confining potentials in the plasma. The phenomenological potentials are in principle two, one which binds the $q\bar{q}$ pair into a meson and one which binds the qqq triplet into a baryon. For the first one, we can use the potential by which the spectrum of the $c\bar{c}$ system has been studied [3]; for the qqq system the potential will be chosen using the *diquark* model [8] with the requirement that the quark–diquark binding force be the same as the quark–antiquark force. The final outcome is very different from the previous results: a complicated

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shielding and antishielding behavior is found. The space integral of the correlation functions yields, however, a more transparent result. There is, in this case, a clear difference between the qq system and the $q\bar{q}$ system. After the final section with the conclusions, two appendices present some details of the calculations needed.

2 General aspects of the treatment

2.1 Review of the formalism

We begin by reviewing the usual treatment of the electromagnetic plasma in terms of correlation functions and extending it to the chromodynamical situation, characterized by the presence of non-commuting charges. One starts from the definition of the canonical partition function

$$Z = \frac{Z_0}{V^N} \int \mathrm{e}^{-\beta U}\, \mathrm{d}^{3N} r, \qquad U = \sum_{i<j} u_{ij}(r_{ij}).$$

The interaction is given by the sum of central two-body potentials and its behavior at $r_{ij} \to 0$ must be regular enough to avoid a divergence of the partition function. The integrand of Z can be expanded in multiple correlations as

$$\mathrm{e}^{-\beta U} = \prod_l \mathcal{D}(\mathbf{r}_l) + \sum_{i<j} \mathcal{C}^{(2)}(\mathbf{r}_i, \mathbf{r}_j) \prod_{l \neq i,j} \mathcal{D}(\mathbf{r}_l) + \sum_{i<j<k} \mathcal{C}^{(3)}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) \prod_{l \neq i,j,k} \mathcal{D}(\mathbf{r}_l) + \dots .$$

For a uniform plasma, the one-body distribution is a constant:
$\mathcal{D}(\mathbf{r}_l) = (Z/Z_0)^{1/N}$. The many-body functions $\mathcal{C}^{(2)}, \mathcal{C}^{(3)}, \dots$ are defined as pure correlations, i.e. $\int \mathcal{C}^{(J)}\, \mathrm{d}^3\mathbf{r}_J = 0$. It is useful to renormalize the functions $\mathcal{C}^{(J)} = (Z/Z_0)^{J/N} C^{(J)}$, so that the expressions of the many-body distributions become

$$W_2(\mathbf{r}_i, \mathbf{r}_j) = \frac{Z_0}{Z V^{N-2}} \int \mathrm{e}^{-\beta U} \prod_{l \neq i,j} \mathrm{d}r_l = [1 + C^{(2)}(\mathbf{r}_i, \mathbf{r}_j)], \tag{1}$$

$$W_3(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) = 1 + [C^{(2)}(\mathbf{r}_i, \mathbf{r}_j) + C^{(2)}(\mathbf{r}_i, \mathbf{r}_k) + C^{(2)}(\mathbf{r}_j, \mathbf{r}_k)] + C^{(3)}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) \tag{2}$$

and so on. Using then the expressions for W_2, W_3, one obtains for $C \equiv C^{(2)}$ the following equation:

$$\frac{\partial C(\mathbf{r}_1, \mathbf{r}_2)}{\partial r_{1,v}} = -\beta \left(\frac{\partial u_{12}}{\partial r_{1,v}} + \frac{1}{V} \sum_{l \neq 1,2} \int \mathrm{d}^3 r_l \left[\frac{\partial u_{12}}{\partial r_{1,v}} C(\mathbf{r}_l, \mathbf{r}_2) \right] \right), \quad v = x, y, z. \tag{3}$$

In the non-commutative case the formal definition of the canonical partition function is the same; however, since there is a matrix structure in color space, a matrix multiplication is implied and a trace must be taken. The integrand of the partition function is expanded into multiple correlations as in the commutative case and the plasma is assumed uniform in space and also isotropic in color. The one-particle distribution is in this way constant in space and diagonal in the color indices $\mathcal{D}(q_i) = R$. The functions $\mathcal{C}^{(J)}$ are again defined as the pure correlations of order J and we redefine them as $\mathcal{C}^{(J)} = R^J C^{(J)}$. So for the two-body distribution we still have an expansion as before, with W and C, which are matrices in color space. Now we want to find an equation for the two-body distribution $W(q_i, q_j)$. In general, the derivative of U will not commute with U, because they are matrices, so we use the representation

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathrm{e}^A = \int_0^1 \mathrm{e}^{xA} \frac{\mathrm{d}A}{\mathrm{d}t} \mathrm{e}^{(1-x)A}\, \mathrm{d}x,$$

which may be verified by comparing the series expansion of both sides.

Identifying now A with $-\beta U$ and defining $\tau = x\beta$, distributions at different temperatures come into play (the presence of integrations over the inverse temperature is a well-known feature of quantum statistics [4]); in this way the variable τ appears in the function C_τ. The equation for the two-body correlation is then

$$\frac{\partial C_\beta(\mathbf{r}_1, \mathbf{r}_2)}{\partial r_{1,v}} = -\int_0^\beta \mathrm{d}\tau \left(\frac{\partial u_{12}}{\partial r_{1,v}} + \frac{1}{3V} \sum_{l \neq 1,2} \int \mathrm{d}^3 r_l \times \left[\frac{\partial u_{12}}{\partial r_{1,v}} C_{\beta-\tau}(\mathbf{r}_l, \mathbf{r}_2) + C_\tau(\mathbf{r}_l, \mathbf{r}_2) \frac{\partial u_{12}}{\partial r_{1,v}} \right] \right).$$

After taking the second derivative with respect to r_1, because of the presence of an integration in the inverse temperature τ, the Laplace transform with respect to β (conjugate variable s) is performed; owing to a more general form of the potential, it is convenient to perform the Fourier transform with respect to space (conjugate variable $\mathbf{k}$). With these transformations, the result takes the form

$$-k^2 \check{C}(s; \mathbf{k}) = \frac{T(\mathbf{k})}{s^2} + \frac{1}{3Vs} \Big[\sum_{l \neq 1,2} T(\mathbf{k}) \check{C}(s; \mathbf{k}) + \check{C}(s; \mathbf{k}) T(\mathbf{k}) \Big]. \tag{4}$$

Both $\check{C}(s; \mathbf{k})$ and $T(\mathbf{k})$ are color matrices; $T(\mathbf{k})$ is in particular independent of $\mathbf{k}$ in the case of the Coulomb potential. Before proceeding to the explicit color structure, attention is focused on the thermal distribution of quarks and gluons.

2.2 Densities of quarks, antiquarks and gluons

When we examine the thermal production of gluons, we cannot ignore the concurrent production of quark–antiquark pairs. We investigate briefly the problem in this form. Assuming the baryonic density, $b = \rho - \bar{\rho}$, i.e. the quark density minus the antiquark density, as given, one finds the simultaneous production of gluons and of quark–antiquark pairs. When the quark mass is neglected or is given as a small correction, the expression for the baryonic density is simple and it has the following form [4]:

$$b = \frac{g_f}{2\,\pi^2} \int_0^\infty \mathrm{d}\epsilon \left(\epsilon^2 - \frac{1}{2} m^2\right) \left[\frac{1}{\mathrm{e}^{\beta\,(\epsilon-\mu)}+1} - \frac{1}{\mathrm{e}^{\beta\,(\epsilon+\mu)}+1}\right]. \tag{5}$$

In particular, the antiquark density is given by

$$\bar\rho = \frac{g_f}{2\,\pi^2} \int_0^\infty \frac{\mathrm{d}\epsilon\,(\epsilon^2-\frac{1}{2}m^2)}{1+\mathrm{e}^{\beta\,(\epsilon+\mu)}} = -\frac{g_f}{2\,\pi^2}\left[\frac{2}{\beta^3}\,\mathcal{L}_3\left(-\mathrm{e}^{-\beta\,\mu}\right) + \frac{m^2}{2\beta}\,\ln(1+\mathrm{e}^{-\beta\,\mu})\right],$$

where $\mathcal{L}_k(z) = \sum_{n=1}^{\infty} z^n/n^k$ and for the weight we use $g_f = 12$, resulting from a factor 2 for the spin, a factor 3 for the color and a factor 2 for the flavor because we consider only u and d quarks.

From (5) it follows that

$$b = \frac{g_f}{6\,\pi^2}\,\mu^3 + \frac{g_f}{6\,\beta^2}\,\mu - \frac{g_f}{4\,\pi^2} m^2\,\mu.$$

From this relation we can find the conditions in which the mass term is negligible: either $m \ll 1/\beta$ or $m \ll \mu$. From now on we shall assume that at least one of these conditions is fulfilled and the mass term is dropped. Solving the previous equation for μ, we find

$$\beta\,\mu = 3^{-\frac{2}{3}}\left[\left(\frac{27\,\pi^2\,x}{g_f}\right)\left(1+\sqrt{1+\left(\frac{\pi\,g_f}{9\,\sqrt{3}\,x}\right)^2}\right)\right]^{\frac{1}{3}} - 3^{-\frac{1}{3}}\,\pi^2\left[\left(\frac{27\,\pi^2\,x}{g_f}\right)\left(1+\sqrt{1+\left(\frac{\pi\,g_f}{9\,\sqrt{3}\,x}\right)^2}\right)\right]^{-\frac{1}{3}}$$

with $x = b\,\beta^3$. In this way we obtain the actual expression for $\bar\rho$ in terms of b and β; the total density which appears in the previous formulae is given by $n = b + 2\bar\rho$. It is now possible to give a semi-quantitative analysis of the different densities which are relevant for our problem: we start by assigning a density b which we take equal to $2\,\mathrm{fm}^{-3}$. In Fig. 1, Fig. 2 and Fig. 3 the values of $\beta\mu$, n and the ratio γ/n as functions of $1/\beta$ are shown. We have worked out three cases: the first is chosen so that the density of antiquarks is much less than b, we take $1/\beta = 150\,\mathrm{MeV}$, this gives $\bar\rho \approx \frac{1}{20}\,b$ and for the corresponding gluon density $\gamma/n = 0.40$. In the second case we take $1/\beta = 300\,\mathrm{MeV}$, this gives $\bar\rho \approx \frac{3}{2}\,b$ and $\gamma/n = 0.84$. In the third case $1/\beta = 450\,\mathrm{MeV}$ is considered, which produces $\bar\rho \approx 6b$ and $\gamma/n = 0.88$, which is the 'saturation' value for the γ/n ratio. In these expressions the density of gluons has been taken as the free-boson thermal density, which amounts to

$$\frac{g_b}{2\pi^2}\,\frac{2}{\beta^3}\,\mathcal{L}_3(1).$$

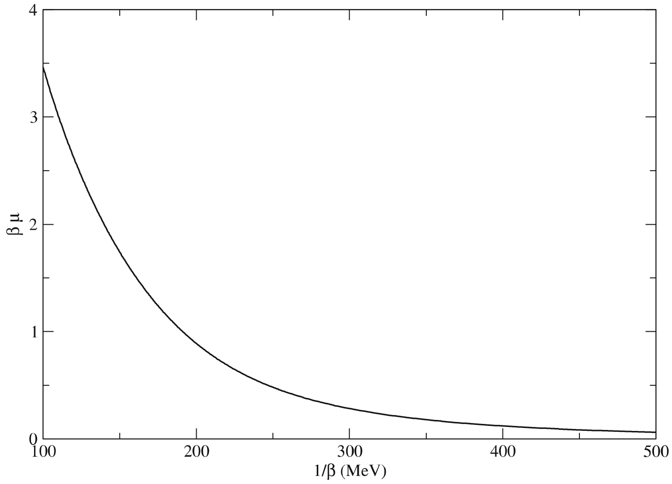

Fig. 1. $\beta\mu$ as a function of $1/\beta$ with initial quark density given by $b = 2\,\mathrm{fm}^{-3}$; note that it is an adimensional quantity

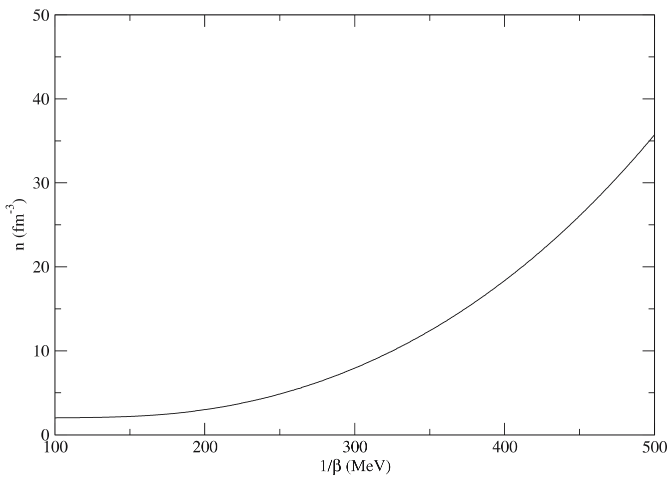

Fig. 2. Fermion density n as a function of $1/\beta$ with initial quark density given by $b = 2\,\mathrm{fm}^{-3}$

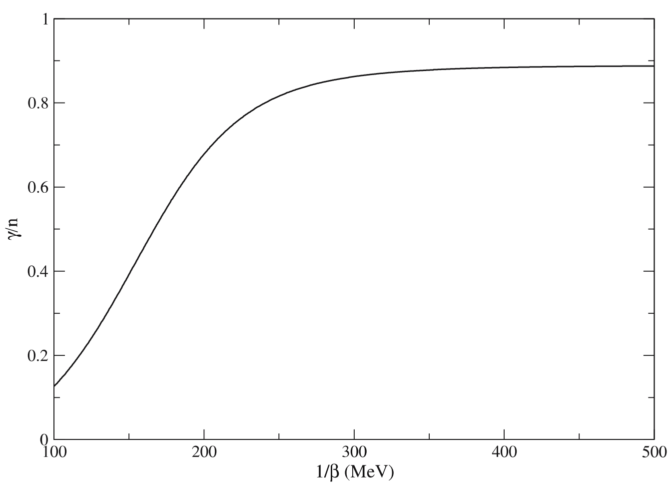

Fig. 3. γ/n ratio as a function of $1/\beta$ with initial quark density given by $b = 2\,\mathrm{fm}^{-3}$

Here $g_b = 16$; i.e. a factor 2 for the spin and a factor 8 for the color. Since we have $\beta\mu \to 0$ for $\beta \to 0$, in that limit

$$\frac{\gamma}{n} = \frac{g_b}{2g_f} \frac{\mathcal{L}_3(1)}{-\mathcal{L}_3(-1)} = \frac{8}{9}$$

and this limit is numerically reached already at $1/\beta = 390$ MeV. It is clear that this ratio could be modified by the inclusion of the gluon self-interaction; in this case, moreover, the fermion population could also be modified by production and annihilation processes.

3 Effect of thermal gluons

Since we are interested in the effect of the interaction at relatively large distance, we choose those terms which present a singularity in the t-channel, as was already done for the qq and $q\bar{q}$ cases. Some details of the proposed dynamics are given in Appendix A. In the qg system this means considering a quark and a gluon interacting through the exchange of a virtual gluon. The calculations shown below say that the introduction of this term gives a definite modification of the shielding effect; the resulting shape is however not very different from the case where only quarks and antiquarks are present. For this reason the evaluation of the effect of the gg system, which could be done with the same procedures here presented, is not performed.

Now we describe the actual calculations. The factors $\check{C}(s;k)$ and T of (4) are matrices; hence, their order is relevant. In order to proceed it is necessary to state the color structure of the quarks and of the gluons, so the indices will now have to be displayed and the precise meaning of T and $\check{C}$ has to be specified. It is not convenient to work with two set of indices, for quark and gluon respectively, so the color charge of the gluon is represented by a traceless tensor with triplet indices, like A^c_d with $A^n_n = 0$.

The term T is specified in this way: the interactions qq and $\bar{q}\bar{q}$ are given by

$$I^{b,d}_{a,c} = \frac{1}{2}\left[\delta^d_a\,\delta^b_c - \frac{1}{3}\,\delta^b_a\,\delta^b_c\right],$$

the interaction $q\bar{q}$ is given by $-I$; the interaction qg is given by

$$J^{b,dg}_{a,cf} = \frac{1}{2\,\mathrm{i}}\left[\delta^d_a\,\delta^b_c\,\delta^g_f - \delta^g_a\,\delta^d_c\,\delta^b_f\right],$$

the interaction $\bar{q}g$ is given by $-J$.

The term $\check{C}$ is indicated with different symbols following its quark and gluon content and is decomposed according to the color structure: for qq and $\bar{q}\bar{q}$ it is called respectively Q and $\bar{Q}$ and decomposed in terms of *triplet* and *sextet*, for $q\bar{q}$ it is called M and decomposed into *singlet* and *octet* while for qg or $\bar{q}g$ it is called B or $\bar{B}$ and decomposed into *triplet*, *sextet* and *15-plet* by means of the following projectors:

$$^3\Pi^{b,d}_{a,c} = \frac{1}{2}\left[\delta^b_a\,\delta^d_c - \delta^d_a\,\delta^b_c\right], \quad {}^6\Pi^{b,d}_{a,c} = \frac{1}{2}\left[\delta^b_a\,\delta^d_c + \delta^d_a\,\delta^b_c\right],$$

$$^1\Pi^{b,d}_{a,c} = \frac{1}{3}\,\delta^d_a\,\delta^d_a, \quad {}^8\Pi^{b,d}_{a,c} = \left[\delta^b_a\,\delta^d_c - \frac{1}{3}\,\delta^d_a\,\delta^b_c\right],$$

$$^3P^{b,dg}_{a,cf} = \frac{3}{8}\,\delta^g_a\,\delta^d_c\,\delta^b_f - \frac{1}{8}\left[\delta^d_a\,\delta^g_c\,\delta^b_f + \delta^g_a\,\delta^b_c\,\delta^d_f\right] + \frac{1}{24}\,\delta^b_a\,\delta^g_c\,\delta^d_f,$$

$$^6P^{b,dg}_{a,cf} = \frac{1}{2}\left[\delta^b_a\,\delta^d_c - \delta^d_a\,\delta^b_c\right]\delta^g_f - \frac{1}{4}\left[\delta^g_a\,\delta^d_c\,\delta^b_f + \delta^b_a\,\delta^g_c\,\delta^d_f - \delta^g_a\,\delta^b_c\,\delta^d_f - \delta^d_a\,\delta^g_c\,\delta^b_f\right],$$

$$^{15}P^{b,dg}_{a,cf} = \frac{1}{2}\left[\delta^b_a\,\delta^d_c + \delta^d_a\,\delta^b_c\right]\delta^g_f - \frac{1}{8}\left[\delta^g_a\,\delta^d_c\,\delta^b_f + \delta^b_a\,\delta^g_c\,\delta^d_f + \delta^g_a\,\delta^b_c\,\delta^d_f + \delta^d_a\,\delta^g_c\,\delta^b_f\right].$$

The correct normalization of the projector is easily verified:

$$^1\Pi^{f,g}_{f,g} = 1, \quad {}^8\Pi^{f,g}_{f,g} = 8, \quad {}^3\Pi^{f,g}_{f,g} = 3, \quad {}^6\Pi^{f,g}_{f,g} = 6,$$
$$^3P^{f,g,h}_{f,g,h} = 3, \quad {}^6P^{f,g,h}_{f,g,h} = 6, \quad {}^{15}P^{f,g,h}_{f,g,h} = 15.$$

For the different amplitudes, the equations corresponding to the general form of (4) are

$$\begin{aligned}
-k^2\,Q^{b,d}_{a,c} &= \frac{4\,\pi\,\alpha}{s^2}\,I^{b,d}_{a,c} + \frac{4\,\pi\,\alpha}{s}\,\Big[\Big(Q^{b,g}_{a,f}\,I^{f,d}_{g,c} + I^{b,g}_{a,f}\,Q^{f,d}_{g,c}\Big)\,\rho \\
&\quad + \Big(M^{b,g}_{a,f}\,\Big(-I^{f,d}_{g,c}\Big) + \Big(-I^{b,g}_{a,f}\Big)\,\bar{M}^{f,d}_{g,c}\Big)\,\bar{\rho} \\
&\quad + \Big(B^{b,gl}_{a,fm}\,J^{d,fm}_{c,gl} + J^{b,gl}_{a,fm}\,B^{d,fm}_{c,gl}\Big)\,\gamma\Big], \\
-k^2\,M^{b,d}_{a,c} &= -\frac{4\,\pi\,\alpha}{s^2}\,I^{b,d}_{a,c} + \frac{4\,\pi\,\alpha}{s} \\
&\quad \times\Big[\Big(Q^{b,g}_{a,f}\,\Big(-I^{f,d}_{g,c}\Big) + I^{b,g}_{a,f}\,M^{f,d}_{g,c}\Big)\,\rho \\
&\quad + \Big(M^{b,g}_{a,f}\,I^{f,d}_{g,c} + (-I^{b,g}_{a,f})\,\bar{Q}^{f,d}_{g,c}\Big) \\
&\quad \bar{\rho} + \Big(B^{b,gl}_{a,fm}\,(-J^{d,fm}_{c,gl}) + J^{b,gl}_{a,fm}\,\bar{B}^{d,fm}_{c,gl}\Big)\,\gamma\Big],
\end{aligned} \tag{6}$$

$$\begin{aligned}
-k^2\,B^{b,dg}_{a,cf} &= \frac{4\,\pi\,\alpha}{s^2}\,J^{b,dg}_{a,cf} + \frac{4\,\pi\,\alpha}{s} \\
&\quad \times\Big[\Big(Q^{b,m}_{a,l}\,J^{l,dg}_{m,cf} + I^{b,m}_{a,l}\,B^{l,dg}_{m,cf}\Big)\,\rho \\
&\quad + \Big(M^{b,m}_{a,l}\,(-J^{l,dg}_{m,cf}) + (-I^{b,m}_{a,l})\,\bar{B}^{l,dg}_{m,cf})\Big)\,\bar{\rho}\Big].
\end{aligned}$$

The coefficients ρ, $\bar{\rho}$, γ give the densities of quarks, antiquarks and gluons; the equations for $\bar{Q}$ and $\bar{B}$ can be obtained by interchanging $Q \Leftrightarrow \bar{Q}$, $\rho \Leftrightarrow \bar{\rho}$ and $B \Leftrightarrow -\bar{B}$ in the equations for Q and B. It is possible and useful to give a graphical representation to (6), see Fig. 5, through the rules set out in Fig. 4; it is consistent with the color structure to use lower indices for incoming quarks and outgoing antiquarks and upper indices for incoming antiquarks and outgoing quarks. Since the gluon color structure is given in the spinorial version (Appendix A), the gluon is represented in the form of a $q\bar{q}$ pair, where up-arrow and down-arrow are used respectively for quarks and antiquarks. We recall that the interaction term J is pure imaginary. In the last equation of (6) it can be seen that B depends linearly on J. In the equations for M and Q, however, B and J appear always together and always multiplied by γ, so it is

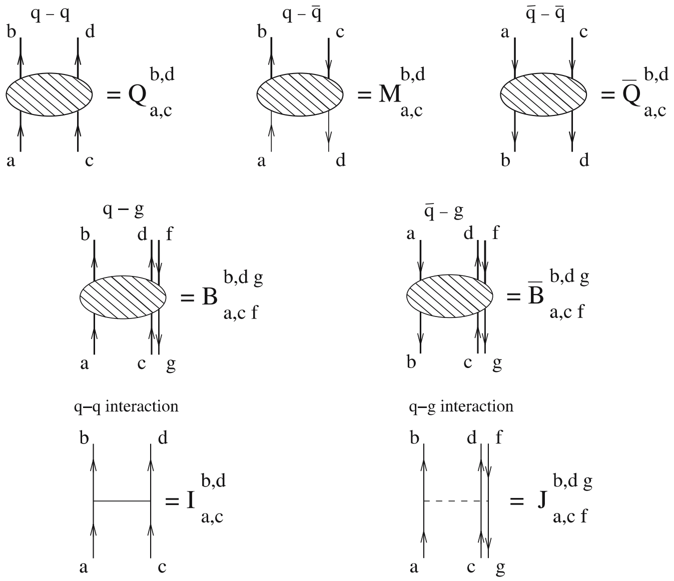

Fig. 4. Rules for a graphical interpretation of (6): *up and down arrows* indicate respectively quarks and antiquarks while the gluons are represented in the form of a $q\bar{q}$ pair (*up–down arrow*)

useful to perform the substitutions $J = -\mathrm{i}J'$, $B = -\mathrm{i}B'$. In this way in the last equation we obtain a trivial factor $-\mathrm{i}$, in the other two we find a change of sign in the terms that multiply γ. In the whole set of equations (6) this is performed by eliminating the 'prime' in J and B and changing the sign $\gamma \to -\gamma$. When we perform the decomposition into amplitudes of fixed color representation

$$\begin{aligned} M &= {}^1\Pi\, F_1 + {}^8\Pi\, F_8, \\ Q &= {}^3\Pi\, F_3 + {}^6\Pi\, F_6, \\ \bar{Q} &= {}^3\Pi\, F_3 + {}^6\Pi\, \bar{F}_6, \\ B &= {}^3P\, B_3 + {}^6P\, B_6 + {}^{15}P\, B_{15}, \\ \bar{B} &= {}^3P\, \bar{B}_3 + {}^6P\, \bar{B}_6 + {}^{15}P\, \bar{B}_{15} \end{aligned}$$

we find, contracting the interaction term J with the projectors P, that the right-hand sides of (6) for the $(qq,\, q\bar{q}, \bar{q}\bar{q})$ systems contain only the $I_{a,c}^{b,d}$ tensor; in fact in any case an octet is exchanged in the t-channel. In this way the relations

$$F_8 = -\frac{1}{8}\,F_1, \quad F_6 = -\frac{1}{2}\,F_3, \quad \bar{F}_6 = -\frac{1}{2}\,\bar{F}_3 \tag{7}$$

hold as in the case without gluons. From the tedious calculations presented in Appendix A, it follows that the expression $F_1 = F_3 + \bar{F}_3$ is still verified with

$$\begin{aligned} F_3 = \bar{F}_3 &= F_3^{(0)} \\ &+ \frac{64\,\pi^2\,\alpha^2\,\gamma}{s\,(k^4\,s^2 + 6\,k^2\,s\,\pi\,\alpha\,n + 48\,\pi^2\,\alpha^2\,\gamma\,n + 8\,\pi^2\,\alpha^2\,n^2)} \\ &\times \left[1 - \frac{2\,\pi\,\alpha\,n}{k^2\,s + 4\,\pi\,\alpha\,n}\right] \end{aligned} \tag{8}$$

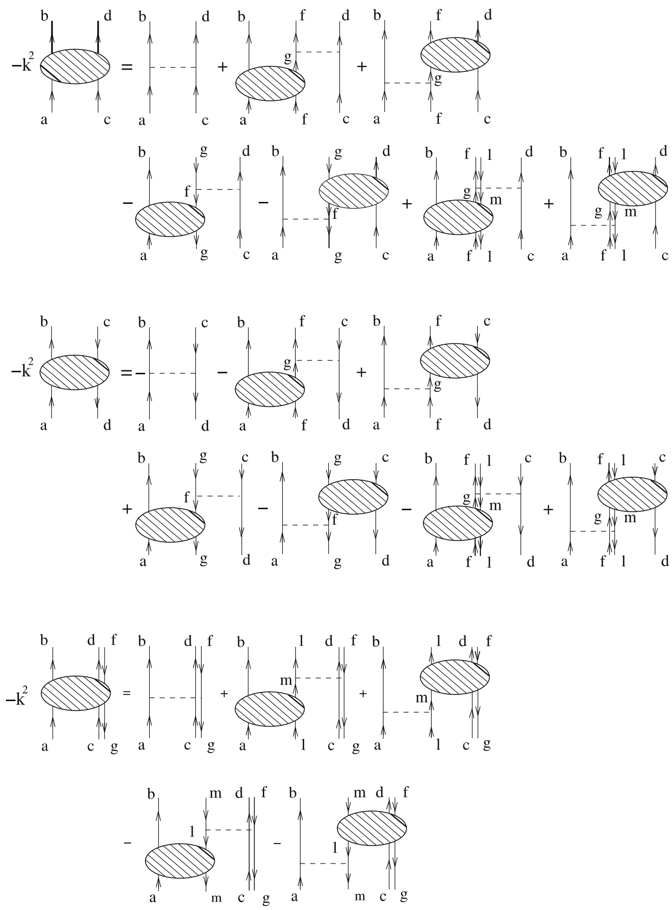

Fig. 5. Graphical representation of (6)

and

$$F_3^{(0)} = \frac{8\,\pi\,\alpha}{3\,s\,(k^2\,s + 4\,\pi\,\alpha\,n)};$$

as it appears from its expression, $F_3^{(0)}$ is the amplitude in the absence of gluons. All the $q\bar{q}$ correlations may be expressed in term of F_3, so we evaluate its Laplace antitransform with the following result:

$$\begin{aligned} \hat{G}_\beta(k^2) &= \frac{2}{3\,n}\left[1 + \frac{6\,\gamma/n}{1 + 6\,\gamma/n}\right] \\ &\times \left[1 - \mathrm{e}^{-\frac{3\,\pi\,n\,\alpha\,\beta}{k^2}}\left(\cos\left(\frac{3\,\pi\,n\,\alpha\,\beta\,y}{k^2}\right)\right.\right. \\ &\left.\left. - \frac{1 - 12\,\gamma/n}{3\,y\,(1 + 12\,\gamma/n)}\,\sin\left(\frac{3\,\pi\,n\,\alpha\,\beta\,y}{k^2}\right)\right)\right], \end{aligned} \tag{9}$$

where $y = \frac{1}{3}\sqrt{48\,\gamma/n - 1}$. The inversion of the Fourier transform leads to an expression that is scarcely transparent, it is reported for completeness in Appendix A; the correlation function in real space has been computed numerically, by means of standard integration subroutines [5],

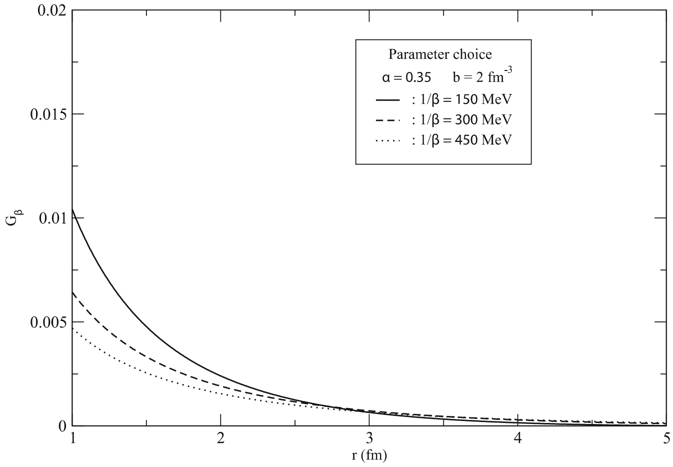

Fig. 6. Correlation function $G_\beta(r)$ in absence of gluons for different choices of temperatures, at fixed coupling constant $\alpha = 0.35$ and initial quark density $b = 2\,\mathrm{fm}^{-3}$

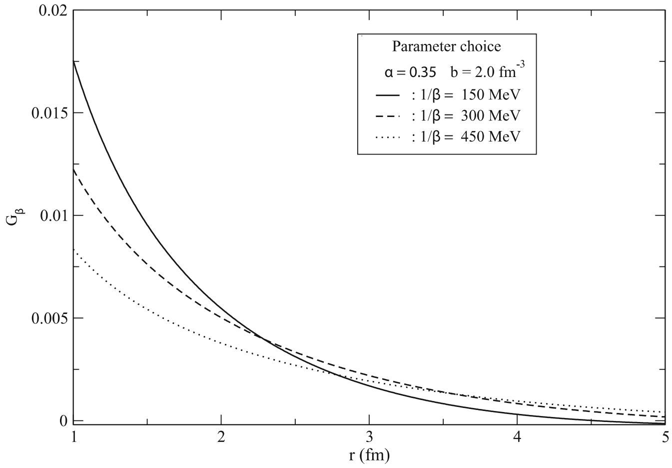

Fig. 7. Correlation function $G_\beta(r)$ in presence of gluons for different choices of temperatures, at fixed coupling constant $\alpha = 0.35$ and initial quark density $b = 2\,\mathrm{fm}^{-3}$

starting from the following integral:

$$G_\beta(r^2) = \frac{1}{2\,\pi^2\,r}\int_0^\infty \mathrm{d}k\,k\,\sin(k\,r)\,\hat{G}_\beta(k^2).$$

In Fig. 6 and Fig. 7 the correlation function without and with gluons for different choices of temperature, at fixed coupling constant $\alpha = 0.35$ [6] and initial quark density $b = 2\,\mathrm{fm}^{-3}$, is shown. In both cases it can be noted that the damping of the correlation, if not exponential, differs very little from that behavior; there is also a long-distance r region (with $r \sim 6$ fm at $1/\beta = 350$ MeV), where the correlation begins to oscillate, but the amplitude of these oscillations is very small, being present when the correlation has already been much damped. The previous plots give a good overall description of the behavior of the correlation function but do not show immediately how these functions differ from a Yukawa shape $\mathrm{e}^{-\mu\,r}/r$. In order to give a more complete description of this property in Fig. 8, the expression $-\ln(r\,G_\beta(r))$ is plotted. Were the Yukawa shape exact, we would find a straight line; these plots confirm that the gluons make the damping weaker, moreover the correlation function deviates from a Yukawa shape a little more than in the absence of gluons. It can be seen that the effect associated with the gluon presence is the production, at fixed temperature, of a perceptible increase in the value of the correlation and consequently a displacement towards higher r values of the region where the correlation functions begin to oscillate.

We may consider, also in view of the comparison with the results presented in the next section, a sort of global correlation effect by taking the integral over r. The following expression is obtained:

$$\int \mathrm{d}^3 r\,\hat{G}_\beta(r^2) = \frac{2}{3\,n}\left[1 + \frac{6\,\gamma/n}{1 + 6\,\gamma/n}\right], \qquad (10)$$

which is just the expression in (9) taken for $k \to 0$, as it must be. What is remarkable about this expression is that there is no explicit dependence on the temperature; nevertheless, a dependence is implicit in the evolution of the densities n and γ as previously shown. As a general comment, we may say that the actual dependence of the gluon density on the temperature is, for our treatment, an ex-

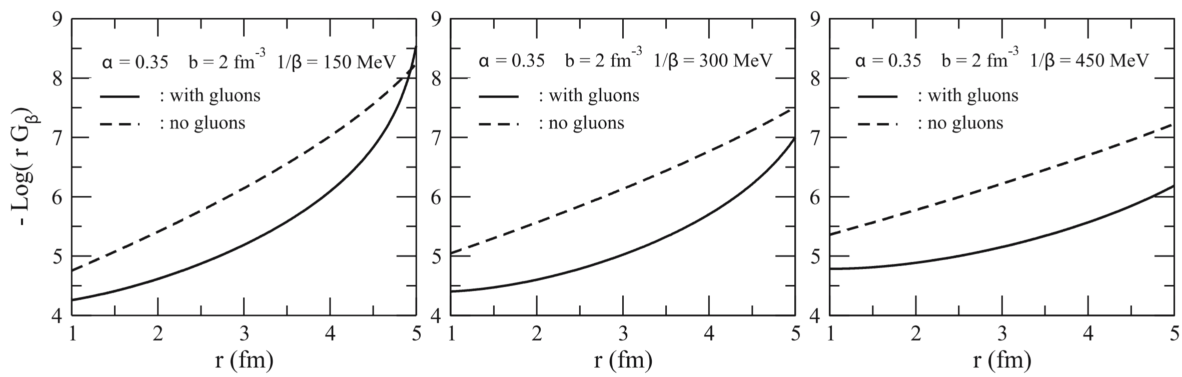

Fig. 8. Minus logarithm of $r\,G_\beta(r)$ with (*solid line*) and without (*dashed line*) curves for different choices of temperature, coupling constant $\alpha = 0.35$ and initial quark density $b = 2\,\mathrm{fm}^{-3}$

ternal input. If, for instance, the dependence of γ on β were different, the quantitative result would change but the main qualitative feature would not.

4 Effects of confinement

In order to study the case in which the confining term plays a fundamental role, we must redefine our starting point. The relation between a local field theory and a confined dynamics is not wholly elucidated in the continuum, although relevant steps in that direction may be given [7]. The confinement must still be treated in a phenomenological way. Within this frame there is more than one option: the two extremes may be described in this way. On one hand we could use a bag model where the confinement is simply provided by the pressure external to the bag; inside the quarks and gluons follow a dynamics that can be described in perturbative form. In such a situation the local dynamics is not very far from the one described in the previous section, so we do not go further in this direction. The other possibility, that will be considered here, is very different from the previous one: quarks and gluons are individually subjected to confining forces which are of the same kind as those acting inside single hadrons. So, we have two kinds of confining potentials, one binding a $q\bar{q}$ pair into an uncolored meson and the other binding a triplet of quarks into an uncolored baryon. The first case is simpler: we must project the singlet color state in the s-channel and then give an explicit form to the confining term. The usual shape for this potential is spherically symmetric, linearly rising with the distance: $u = r/\ell^2$. This form has been long used in studying the meson spectroscopy [3], in particular the bound state of a heavy $q\bar{q}$ pair; in this way a quantitative determination of ℓ is also found. It is useful to remember that the usual interpretation of the constant force is made in terms of a color flux string that eventually breaks down giving rise to a new $q\bar{q}$ pair. For the coming treatment the distance at which the string breaks is not relevant, provided that this happens inside the plasma. In the second case we should introduce a three-body potential of the form

$$v^{abc}_{lmn}(r_1, r_2, r_3) = \epsilon^{abc}\epsilon_{lmn} f(r_1 - r_2, r_1 - r_3).$$

Working with the full three-body potential would lead us to introduce a four-body correlation, to be then reduced to lower-order correlations; this would lead to a form analogous to but more complicated than (1) and (2). We have also more uncertainty in the form of the complete potential, so we decide to work with a two-body potential obtained from the often-used quark–diquark model [8, 9]. This means that we perform the summation over the coordinates of the third quark, we sum over the color index $c = n$ and then integrate in $\mathrm{d}r_3$ over the size of a diquark subsystem. The potential we obtain in this way represents the quark–diquark binding potential. In the quoted interpretation of the potential as an effect of a color flux, the flux between quark and diquark is the same as between quark and antiquark, so the total interaction should be the same. We have only to note that we could have chosen either the quark '2' or the quark '3' as representative of the diquark system; in order to take care of this fact the qq potential is assumed to be one-half of the $q\bar{q}$ potential.

Since this section is devoted to the study of the confinement, we neglect all other interactions that have already been examined. The answers we shall find at the end are very different from those were obtained in the previous cases.

Now we write the equation for the two-body correlations in the presence of confining potentials:

$$\begin{aligned}
-k^2 M^{b,d}_{a,c} &= -\frac{8\pi}{s^2 k^2 l_2^2}\, {}^1\Pi^{b,d}_{a,c} - \frac{8\pi}{s\,k^2} \\
&\quad \times \left[\left(\frac{{}^3\Pi^{b,g}_{a,f}}{9\,l_3^2} M^{f,d}_{g,c} + Q^{b,g}_{a,f} \frac{{}^1\Pi^{f,d}_{g,c}}{l_2^2} \right) \rho \right. \\
&\quad \left. + \left(\frac{{}^1\Pi^{b,g}_{a,f}}{l_2^2} \bar{Q}^{f,d}_{g,c} + M^{b,g}_{a,f} \frac{{}^3\Pi^{f,d}_{g,c}}{9\,l_3^2} \right) \bar{\rho} \right], \\
-k^2 Q^{b,d}_{a,c} &= -\frac{8\pi}{9\,s^2 k^2 l_3^2}\, {}^3\Pi^{b,d}_{a,c} - \frac{8\pi}{s\,k^2} \\
&\quad \times \left[\left(\frac{{}^3\Pi^{b,g}_{a,f}}{9\,l_3^2} Q^{f,d}_{g,c} + Q^{b,g}_{a,f} \frac{{}^3\Pi^{f,d}_{g,c}}{9\,l_3^2} \right) \rho \right. \\
&\quad \left. + \left(\frac{{}^1\Pi^{b,g}_{a,f}}{l_2^2} M^{f,d}_{g,c} + M^{b,g}_{a,f} \frac{{}^1\Pi^{f,d}_{g,c}}{l_2^2} \right) \bar{\rho} \right].
\end{aligned} \tag{11}$$

The meaning of Q, $\bar{Q}$ and M is the same as before; the equations for $\bar{Q}$ can be obtained by interchanging $Q \Leftrightarrow \bar{Q}$ and $\rho \Leftrightarrow \bar{\rho}$ in the equation for Q.

Projecting out the previous equations, we obtain the system of equations indicated in Appendix B, (B.1). From the solution of the previous linear system, for $l_3^2 = 2\,l_2^2 = 2\,l^2$, it follows that

$$\begin{aligned}
F_3 - F_6 &= \frac{4\pi}{27\,s\,l^4}\,\frac{3\,l^2 k^4 s - 191\,\pi\bar{\rho}}{(k^4 s + a_1^2)(k^4 s - b_1^2)}, \\
F_3 + 2\,F_6 &= \frac{4\pi}{81\,s\,l^4}\,\frac{9\,l^2 k^4 s + 280\,\pi\bar{\rho}}{(k^4 s + a_2^2)(k^4 s - b_2^2)}, \\
F_1 - F_8 &= \frac{24\,\pi\,k^4}{3\,l^2}\,\frac{1}{(k^4 s + a_1^2)(k^4 s - b_1^2)}, \\
F_1 + 8\,F_8 &= \frac{24\,\pi\,k^4}{3\,l^2}\,\frac{1}{(k^4 s + a_2^2)(k^4 s - b_2^2)},
\end{aligned} \tag{12}$$

where the expressions for $\bar{F}_3 - \bar{F}_6$ and $\bar{F}_3 + 2\,\bar{F}_6$ can be obtained from the ones of $F_3 - F_6$ and $F_3 + 2\,F_6$ through the substitution $\rho \Leftrightarrow \bar{\rho}$ and

$$\begin{aligned}
a_1^2 &= \frac{2\pi}{9\,l^2}\,(\Delta^{(1)}_{\rho\bar{\rho}} + \rho + \bar{\rho}), \quad b_1^2 = \frac{2\pi}{9\,l^2}\,(\Delta^{(1)}_{\rho\bar{\rho}} - \rho - \bar{\rho}), \\
a_2^2 &= \frac{4\pi}{9\,l^2}\,(\Delta^{(2)}_{\rho\bar{\rho}} - \rho - \bar{\rho}), \quad b_2^2 = \frac{4\pi}{9\,l^2}\,(\Delta^{(2)}_{\rho\bar{\rho}} + \rho + \bar{\rho}),
\end{aligned}$$

where

$$\Delta^{(1)}_{\rho\bar{\rho}} = \sqrt{\rho^2 + 574\,\rho\,\bar{\rho} + \bar{\rho}^2}, \qquad \Delta^{(2)}_{\rho\bar{\rho}} = \sqrt{\rho^2 + 142\,\rho\,\bar{\rho} + \bar{\rho}^2}.$$

We must now invert both the Fourier transform and the Laplace transform; already in performing the first operation, we meet terms with undamped oscillations which are the more typical product of the confining potential: we give here one of such expressions, further details appear in Appendix B.

$$(F_3 - F_6)(r,s) = \frac{1}{4\,\pi\,\Delta^{(1)}_{\rho\bar\rho}} \frac{1}{r\,s^{\frac{3}{2}}} \Bigg[\left(\frac{b_1}{2} - \frac{286\,\pi\,\bar\rho}{9\,b_1\,l^2} \right) \times \left(\cos\left(\sqrt{\frac{b_1}{\sqrt{s}}}\,r \right) - \exp\left(-\sqrt{\frac{b_1}{\sqrt{s}}}\,r \right) \right) + \left(a_1 + \frac{572\,\pi\,\bar\rho}{9\,a_1\,l^2} \right) \exp\left(-\sqrt{\frac{a_1}{2\sqrt{s}}}\,r \right) \times \sin\left(\sqrt{\frac{a_1}{2\sqrt{s}}}\,r \right) \Bigg].$$

In performing the next step, the inversion of the Laplace transform necessary to regain the dependence on temperature, a wild oscillatory behavior of the correlation functions is found, that makes the interpretation difficult. The behavior of the correlation function is anyway given in Fig. 9.

It is more convenient to perform an integration over the spatial variables; since we have the expression for the Fourier transform, the simplest thing to do is to put $\mathbf{k} = 0$ in order to obtain the space integral. However, we have seen that the integrand in $\mathbf{r}$ contains oscillating terms, so it is not evident that this simple treatment is allowed. For this reason the 'dangerous term' is studied in detail in Appendix B by putting in an exponential damping term and finally removing it; we may conclude in this way that the integrated correlation is finite. In detail, we find two different answers. For the amplitude F_3, which gives the qq correlation, the spatial integral is finite and given by

$$\int F_3(r)\;\mathrm{d}^3r \simeq \frac{0.41}{\rho}.$$

This result has to be compared with (10); as in the previous case neither the temperature $1/\beta$ nor the dynamical parameters (l^2, α) enter the result; the temperature enters indirectly through the densities. Contrary to (10), which contains the total fermionic density n, here only the quark density ρ enters (and evidently the antiquark density $\bar\rho$ in the $\bar q\bar q$ correlation). As for the amplitude F_1, which gives the $q\bar q$ correlation, the spatial integral is zero; it has however been remembered that this 'zero' is found in the simplified calculations, where neither the thermal gluons nor the perturbative potential are present.

5 Conclusions

Two aspects of mutual shielding have been examined; the first amounts to the inclusion of thermal gluons in the effect of mutual screening in a $q\bar q$ plasma; it refines a previous analysis given in terms of a pure quark–antiquark population and it confirms the results. In particular, the correlation length and the shape of the damping are still the same for qq and $q\bar q$, so that the shielding effect would be the same in the meson and in the baryon production. The second instance begins with a phenomenological introduction of confining potentials. While the $q\bar q$ interaction is better known, the qq forces present the problem that a real confinement is produced only when three quarks, in suitable color states, are put together; this difficulty has been circumvented by using a quark–diquark model. This model is consistent with the interpretation of the confinement as due to a color string and moreover it links the strength of this interaction to the $q\bar q$ interaction. The result is much more complicated than in the first case. In fact, no punctual shielding has been found, what may raise doubts about the whole treatment. Luckily a spatial integration of the shielding and antishielding effects yields a finite result, so that the final picture we obtain in this way is much more simplified. An oscillating behavior was present also when only the one-gluon potential was considered; it is one of the qualitative differences from the Abelian case, but in this case the oscillations begin when the correlation function is already exponentially small. Conversely, shielding and antishielding effects are produced, even if we consider a confining but Abelian potential [10].

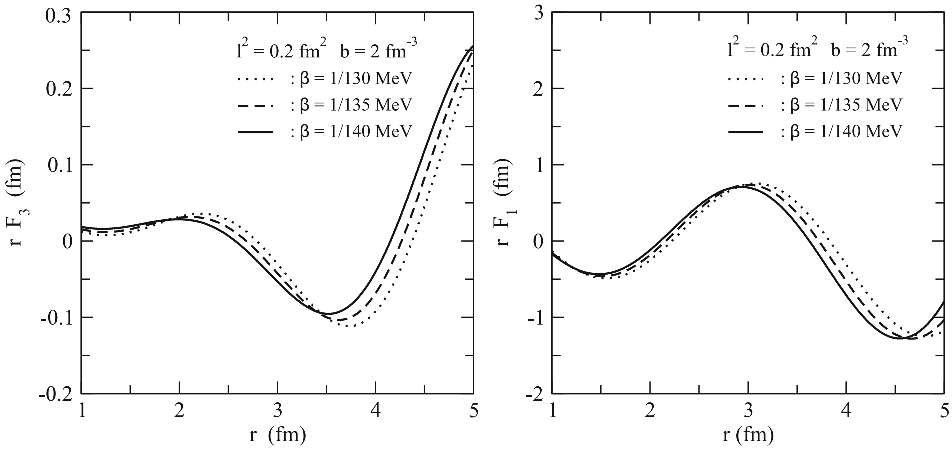

Fig. 9. $r\,F_3$ and $r\,F_1$ for confinement scale $1/\beta$ in a neighborhood of π^0 mass and strength constant $l^2 = 0.2\,\mathrm{fm}^2$ [3]

The shielding effect depends on the temperature. This dependence has two origins: one is the kinematical effect that is present in every plasma-like system, the other is dynamical and typical of a relativistic system, since it comes from the thermal production of particles, both gluons and $q\bar{q}$ pairs. Initially there are more quarks than antiquarks; however, looking at the evolution of the chemical potential (Fig. 1), we see that there is an equivalent temperature, not extremely high, at which the effects of the initial condition become very small. As we mentioned before, in order to obtain the result we used thermal densities both for the quarks and for the gluons. We note that a possible evolution of those densities induced by the elementary QCD vertices, which are strongly local, will not give rise to long-range spatial correlation. An item that requires some consideration is the comparison with other ways of dealing with the same phenomenon. The treatment given in terms of strong coupling on the lattice is difficult to compare with [11–17], because it is very different from the beginning; more similar are the treatments in terms of thermal Green function as in [4]. In that case one extracts the contribution of the pole of the propagator in momentum space, so that a precise Yukawa-like decay of the correlation function is certainly produced. The more unusual result, i.e. the presence of an antishielding, was already foreseen, but in the different situation of an anisotropic plasma [14]. We have presented here a particular and definite model in which some of the dynamical features, the static interactions, are put into evidence and worked out to yield a quantitative result. Some effects that are present in other treatments are lacking, in particular the hydrodynamical aspects [18]: does it also contain something more? Perhaps what is more can be grasped by looking at the structure of the coupled equations (11): the interaction of one particle takes place with other particles, which are already correlated, so that at least at the level of two-fermion distribution, a self- consistent treatment of the correlations is performed. But, also we have seen that this approach allows a common treatment of the perturbative dynamics in terms of single-gluon exchange and confining potential. Although there are very interesting attempts in that direction [7], a common treatment of both processes in terms of a thermal Green function is not yet available.

Appendix A: Solutions with gluons

A.1 Quark–gluon interaction

As previously stated, only the quark–gluon interaction corresponding to a gluon exchange in the t-channel is considered. In the qg interaction there are clearly other terms, like in QED for the Compton scattering. They show singularities either in the s or in the u channel but they are not relevant for the 'long-distance' effects, which we are interested in. The gluon self-interaction contains both the $3g$ vertex, which is the building block of the qg and $\bar{q}g$ terms (B and $\bar{B}$ in Sect. 3) and the $4g$ vertex, which does not yield singularities in the t-channel. For this amplitude the color structure is usually given in mixed form [19] as $f_{ABC}(T_C)^a_b$, where $A,B,C=1,\dots,8$ and $a,b=1,2,3$. This form is not convenient here and will be transformed into a purely spinorial version. Using the fundamental commutator $[T_A,T_B]=\mathrm{i}f_{ABC}T_C$, we express the interaction as a commutator; then defining the matrices with spinorial indices

$$(T^k_l)^a_b=\frac{1}{\sqrt{2}}\left[\delta^a_l\delta^k_b-\frac{1}{3}\delta^a_b\delta^k_l\right]$$

the form of the interactions used in this paper for qq and for qg is

$$I^{b,d}_{a,c}=\frac{1}{2}\left[\delta^d_a\,\delta^b_c-\frac{1}{3}\,\delta^b_a\,\delta^b_c\right],$$
$$J^{b,dg}_{a,cf}=\frac{1}{2\,\mathrm{i}}\left[\delta^d_a\,\delta^b_c\,\delta^g_f-\delta^g_a\,\delta^d_c\,\delta^b_f\right].$$

Note that the antisymmetry of J with respect to the exchange $(c,d)\leftrightarrow(f,g)$ together with the factor i ensures the hermiticity of the interaction.

A.2 Derivation of F_3 correlation

Starting from (7), we obtain the following system of equations:

$$\begin{aligned}
&k^2\,F_1-\frac{16\,\pi\,\alpha}{3\,s^2}+\frac{4\,\pi\,\alpha}{s}\\
&\quad\times\Bigg[F_3\,\rho+\bar{F}_3\,\bar{\rho}+F_1\left(\frac{\rho+\bar{\rho}}{2}\right)\\
&\qquad+2\left(\bar{B}_3+\bar{B}_6-B_3-B_6\right)\,\gamma\Bigg]=0,\\
&k^2\,F_3-\frac{8\,\pi\,\alpha}{3\,s^2}+\frac{4\,\pi\,\alpha}{s}\\
&\quad\times\left[F_3\,\rho+F_1\left(\frac{\bar{\rho}}{2}\right)-2\,(B_3+B_6)\,\gamma\right]=0,\\
&k^2\,\bar{F}_3-\frac{8\,\pi\,\alpha}{3\,s^2}+\frac{4\,\pi\,\alpha}{s}\\
&\quad\times\left[\bar{F}_3\,\bar{\rho}+F_1\left(\frac{\rho}{2}\right)+2\left(\bar{B}_3+\bar{B}_6\right)\,\gamma\right]=0,
\end{aligned}\tag{A.1}$$

where B_{15} and $\bar{B}_{15}$ have been substituted by means of the following relations:

$$B^{x,bd}_{x,ac}=\bar{B}^{x,bd}_{x,ac}=0\Rightarrow\quad \begin{aligned}&3\,B_3+6\,B_6+15\,B_{15}=0,\\ &3\bar{B}_3+6\bar{B}_6+15\,\bar{B}_{15}=0.\end{aligned}$$

The system (A.1) yields immediately $F_1=F_3+\bar{F}_3$ and is reduced to a two-equation system:

$$\begin{aligned}
&F_3\left[k^2+\frac{4\,\pi\,\alpha}{s}\,\left(\rho+\frac{\bar{\rho}}{2}\right)\right]+\bar{F}_3\,\left[\frac{2\,\pi\,\alpha\,\bar{\rho}}{s}\right]\\
&\quad-\frac{8\,\pi\,\alpha}{3\,s^2}-\frac{8\,\pi\,\alpha}{s}\,\gamma\,(B_3+B_6)=0,\\
&F_3\,\left[\frac{2\,\pi\,\alpha\,\rho}{2\,s}\right]+\bar{F}_3\left[k^2+\frac{4\,\pi\,\alpha}{s}\,\left(\bar{\rho}+\frac{\rho}{2}\right)\right]\\
&\quad-\frac{8\,\pi\,\alpha}{3\,s^2}+\frac{8\,\pi\,\alpha}{s}\,\gamma\,(\bar{B}_3+\bar{B}_6)=0.
\end{aligned}$$

From the previous linear system, it follows that F_3 and $\bar{F}_3$ can be expressed in terms of (B_3+B_6) and $(\bar{B}_3+\bar{B}_6)$, as follows:

$$F_3 = F_3^{(0)} + f_3^{(1)}\,(B_3+B_6) + f_3^{(2)}\,(\bar{B}_3+\bar{B}_6) f_3^{(2)}\,(\bar{B}_3+\bar{B}_6), \tag{A.2}$$

where

$$\begin{aligned}
F_3^{(0)} &= \bar{F}_3^{(0)} = \frac{8\,\pi\,\alpha}{3\,s\,(k^2\,s+4\,\pi\,\alpha\,n)},\\
f_3^{(1)} &= \frac{8\,\pi\,\alpha\,\gamma}{k^2\,s+4\,\pi\,\alpha\,n} + \frac{16\,\pi^2\,\alpha^2\,\gamma\,\bar{\rho}}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)},\\
f_3^{(2)} &= \frac{16\,\pi^2\,\alpha^2\,\gamma\,\bar{\rho}}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)},\\
\bar{f}_3^{(1)} &= -\frac{16\,\pi^2\,\alpha^2\,\gamma\,\rho}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)},\\
\bar{f}_3^{(2)} &= -\frac{8\,\pi\,\alpha\,\gamma}{k^2\,s+4\,\pi\,\alpha\,n} - \frac{16\,\pi^2\,\alpha^2\,\gamma\,\rho}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)}
\end{aligned}$$

and $n=\rho+\bar{\rho}$. Furthermore, we obtain

$$F_1 = 2\,F_3^{(0)} + f_1^{(1)}\,(B_3+B_6) + f_1^{(2)}\,(\bar{B}_3+\bar{B}_6),$$

$$\begin{aligned}
f_1^{(1)} &= \frac{8\,\pi\,\alpha\,\gamma\,(k^2\,s+4\,\pi\,\alpha\,\bar{\rho})}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)},\\
f_1^{(2)} &= -\frac{8\,\pi\,\alpha\,\gamma\,(k^2\,s+4\,\pi\,\alpha\,\rho)}{(k^2\,s+4\,\pi\,\alpha\,n)\,(k^2\,s+2\,\pi\,\alpha\,n)}.
\end{aligned}$$

Projecting the equations satisfied by B and $\bar{B}$ with ${}^J P$, the following equations are obtained:

$$\begin{aligned}
&k^2\,B_3 - \frac{6\,\pi\,\alpha}{s^2} + \frac{2\,\pi\,\alpha}{s}\\
&\quad\times\left[\frac{9}{4}\,F_3\,\rho + \frac{9}{8}\,F_1\,\bar{\rho} + (B_3\,\rho - \bar{B}_3\,\bar{\rho})\right] = 0,\\
&k^2\,B_6 - \frac{2\,\pi\,\alpha}{s^2} + \frac{2\,\pi\alpha}{s}\\
&\quad\times\left[\frac{3}{4}\,F_3\,\rho + \frac{3}{8}\,F_1\,\bar{\rho} + (B_6\,\rho - \bar{B}_6\,\bar{\rho})\right] = 0, \qquad \text{(A.3)}\\
&k^2\,B_{15} + \frac{2\,\pi\,\alpha}{s^2} + \frac{2\,\pi\,\alpha}{s}\\
&\quad\times\left[-\frac{3}{4}\,F_3\,\rho - \frac{3}{8}\,F_1\,\bar{\rho} + (B_{15}\,\rho - \bar{B}_{15}\,\bar{\rho})\right] = 0.
\end{aligned}$$

The equation for $\bar{B}_3$, $\bar{B}_6$ can obtained by the interchange $B_i \Leftrightarrow -\bar{B}_i$ for $i=3,6$, $\rho \Leftrightarrow \bar{\rho}$ and $F_3 \Leftrightarrow \bar{F}_3$ in the equations for B_3, B_6. After some tedious calculations, the following two equations for (B_3+B_6) and $(\bar{B}_3+\bar{B}_6)$ variables are obtained:

$$\begin{aligned}
&\left[k^2 + \frac{2\,\pi\,\alpha}{s}\left(3\,f_3^{(1)}\,\rho + \frac{3}{2}\,f_1^{(1)}\,\bar{\rho} + \rho\right)\right](B_3+B_6)\\
&\quad + \left[\frac{2\,\pi\,\alpha}{s}\left(3\,f_3^{(2)}\,\rho + \frac{3}{2}\,f_1^{(2)}\,\bar{\rho} - \bar{\rho}\right)\right](\bar{B}_3+\bar{B}_6)\\
&\quad - \frac{8\,\pi\,\alpha}{s^2} + \frac{6\,\pi\,\alpha}{s}\,F_3^{(0)}\,(\rho+\bar{\rho}) = 0,\\
&\left[\frac{2\,\pi\,\alpha}{s}\left(3\,\bar{f}_3^{(1)}\,\bar{\rho} + \frac{3}{2}\,f_1^{(1)}\,\rho + \rho\right)\right](B_3+B_6)\\
&\quad + \left[-k^2 + \frac{2\,\pi\,\alpha}{s}\left(3\,\bar{f}_3^{(2)}\,\bar{\rho} + \frac{3}{2}\,f_1^{(2)}\,\rho - \bar{\rho}\right)\right](\bar{B}_3+\bar{B}_6)\\
&\quad - \frac{8\,\pi\,\alpha}{s^2} + \frac{6\,\pi\,\alpha}{2\,s}\,F_3^{(0)}\,(\rho+\bar{\rho}) = 0.
\end{aligned}$$

Solving the previous linear system, we have

$$\begin{aligned}
&B_3+B_6\\
&\quad = \frac{8\,\pi\,\alpha\,(k^2\,s+2\,\pi\,\alpha\,n)}{s\,(k^4\,s^2+6\,k^2\,s\,\pi\,\alpha\,n+48\,\pi^2\,\alpha^2\,\gamma\,n+8\,\pi^2\,\alpha^2\,n^2)},\\
&\bar{B}_3+\bar{B}_6\\
&\quad = -\frac{8\,\pi\,\alpha\,(k^2\,s+2\,\pi\,\alpha\,n)}{s\,(k^4\,s^2+6\,k^2\,s\,\pi\,\alpha\,n+48\,\pi^2\,\alpha^2\,\gamma\,n+8\,\pi^2\,\alpha^2\,n^2)}\\
&\quad = -(B_3+B_6).
\end{aligned}$$

Substituting the previous expression into (A.2), we obtain (8).

A.3 Fourier transform inversion

The inversion of the Fourier transform

$$G_\beta(r^2) = \frac{1}{(2\,\pi)^3}\int \mathrm{d}^3k\ \mathrm{e}^{\mathrm{i}\,\mathbf{k}\cdot\mathbf{r}}\,\hat{G}_\beta(k^2)$$

implies a standard angular integration and then an integral over the radial coordinates that gives

$$\begin{aligned}
G_\beta(r^2) = \frac{1}{2\,\pi^2\,r}\Bigg[&g_\beta^1\ {}_0F_2\left(;\frac{1}{2},\ 2;\ \frac{3\,\pi\,n\,\alpha\,\beta\,(1-\mathrm{i}\,y)}{4}\,r^2\right)\\
&+g_\beta^2\ {}_0F_2\left(;\frac{1}{2},\ 2;\ \frac{3\,\pi\,n\,\alpha\,\beta\,(1+\mathrm{i}\,y)}{4}\,r^2\right)\\
&+g_\beta^3\ {}_0F_2\left(;\frac{3}{2},\ \frac{5}{2};\ \frac{3\,\pi\,n\,\alpha\,\beta\,(1-\mathrm{i}\,y)}{4}\,r^2\right)\\
&+g_\beta^4\ {}_0F_2\left(;\frac{3}{2},\ \frac{5}{2};\ \frac{3\,\pi\,n\,\alpha\,\beta\,(1+\mathrm{i}\,y)}{4}\,r^2\right)\Bigg],
\end{aligned} \tag{A.4}$$

where the generalized hypergeometric functions are used:

$${}_pF_q\,(a_1,\ldots,a_p\,;\,b_1,\ldots,b_q\,;\,z) = \sum_{k=0}^{\infty}\frac{(a_1)_k\cdots(a_p)_k}{(b_1)_k\cdots(b_q)_k}\,\frac{z^k}{k!} \tag{A.5}$$

and the coefficients are

$$
\begin{aligned}
g^1_\beta &= \frac{\pi^2\,\alpha\,\beta\,(\mathrm{i}+y)}{2\,y}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\\
&\quad\times\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}-\mathrm{i}\,y\right),\\
g^2_\beta &= \frac{\pi^2\,\alpha\,\beta\,(-\mathrm{i}+y)}{2\,y}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\\
&\quad\times\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}+\mathrm{i}\,y\right),\\
g^3_\beta &= \frac{\sqrt{3\,\pi\,n\,(4\,\pi\,\alpha\,\beta)^3}\,(1-\mathrm{i}\,y)^{\frac{3}{2}}\,r}{12\,(\mathrm{i}\,y)}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\\
&\quad\times\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}-\mathrm{i}\,y\right),\\
g^4_\beta &= \frac{\sqrt{3\,\pi\,n\,(4\,\pi\,\alpha\,\beta)^3}\,(1+\mathrm{i}\,y)^{\frac{3}{2}}\,r}{12\,(-\mathrm{i}\,y)}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\\
&\quad\times\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}+\mathrm{i}\,y\right).
\end{aligned}
\tag{A.6}
$$

The function $G_\beta(r^2)$ is real because it is the sum of two terms with their complex conjugate. Note that for very low values of $1/\beta$, y could become imaginary; in this case all the addenda would be separately real.

Appendix B: Confinement

Starting from (11), we extract the color structures; it turns out that it is more convenient to work with some definite linear combination: F_6-F_3, $F_3+2\,F_6$, F_1-F_8, $F_1+8\,F_8$, $\bar{F}_6-\bar{F}_3$, $\bar{F}_3+2\,\bar{F}_6$, for which the following system of equation holds:

$$
A * F = C, \tag{B.1}
$$

where

$$
A=\begin{pmatrix}
\left(k^2+\frac{8\,\pi\,\rho}{9\,s\,k^2\,l_3^2}\right) & 0 & \left(\frac{32\,\pi\bar{\rho}}{9\,s\,k^2\,l_2^2}\right) & 0 & 0 & 0\\
0 & \left(k^2-\frac{16\,\pi\,\rho}{9\,s\,k^2\,l_3^2}\right) & 0 & -\left(\frac{16\,\pi\bar{\rho}}{9\,s\,k^2\,l_2^2}\right) & 0 & 0\\
\left(\frac{4\,\pi\,\rho}{s\,k^2\,l_2^2}\right) & 0 & \left(k^2+\frac{4\,\pi\,(\rho+\bar{\rho})}{9\,s\,k^2\,l_3^2}\right) & 0 & \left(\frac{4\,\pi\,\bar{\rho}}{s\,k^2\,l_2^2}\right) & 0\\
0 & -\left(\frac{8\,\pi\,\rho}{s\,k^2\,l_2^2}\right) & 0 & \left(k^2-\frac{8\,\pi\,(\rho+\bar{\rho})}{9\,s\,k^2\,l_3^2}\right) & 0 & -\left(\frac{8\,\pi\,\bar{\rho}}{s\,k^2\,l_2^2}\right)\\
0 & 0 & \left(\frac{32\,\pi\rho}{9\,s\,k^2\,l_2^2}\right) & 0 & \left(k^2+\frac{8\,\pi\,\bar{\rho}}{9\,s\,k^2\,l_3^2}\right) & 0\\
0 & 0 & 0 & -\left(\frac{16\,\pi\,\rho}{9\,s\,k^2\,l_2^2}\right) & 0 & \left(k^2-\frac{16\,\pi\,\bar{\rho}}{9\,s\,k^2\,l_3^2}\right)
\end{pmatrix},
$$

$$
F=\begin{pmatrix} F_3-F_6\\ F_3+2\,F_6\\ F_1-F_8\\ F_1+8\,F_8\\ \bar{F}_3-\bar{F}_6\\ \bar{F}_3+2\,\bar{F}_6\end{pmatrix},\qquad
C=\begin{pmatrix} \frac{8\,\pi}{9\,s^2\,k^2\,l_3^2}\\ \frac{8\,\pi}{9\,s^2\,k^2\,l_3^2}\\ \frac{8\,\pi}{s^2\,k^2\,l_2^2}\\ \frac{8\,\pi}{9\,s^2\,k^2\,l_2^2}\\ \frac{8\,\pi}{9\,s^2\,k^2\,l_3^2}\\ \frac{8\,\pi}{9\,s^2\,k^2\,l_3^2}\end{pmatrix}.
$$

The solutions are given by (12). Taking the Fourier transform of those solutions, we obtain

$$
\begin{aligned}
(F_3-F_6)(r,s) &= \frac{1}{4\,\pi\,\Delta^{(1)}_{\rho\bar{\rho}}}\,\frac{1}{r\,s^{\frac{3}{2}}}\left[\left(\frac{b_1}{2}-\frac{286\,\pi\,\bar{\rho}}{9\,b_1\,l^2}\right)\right.\\
&\quad\times\left(\cos\left(\sqrt{\frac{b_1}{\sqrt{s}}}\,r\right)-\exp\left(-\sqrt{\frac{b_1}{\sqrt{s}}}\,r\right)\right)\\
&\quad+\left(a_1+\frac{572\,\pi\,\bar{\rho}}{9\,a_1\,l^2}\right)\exp\left(-\sqrt{\frac{a_1}{2\,\sqrt{s}}}\,r\right)\\
&\quad\left.\times\sin\left(\sqrt{\frac{a_1}{2\,\sqrt{s}}}\,r\right)\right],\\
(F_3+2\,F_6)(r,s) &= \frac{1}{8\,\pi\,\Delta^{(2)}_{\rho\bar{\rho}}}\,\frac{1}{r\,s^{\frac{3}{2}}}\left[\left(\frac{b_2}{2}+\frac{140\,\pi\,\bar{\rho}}{9\,b_2\,l^2}\right)\right.\\
&\quad\times\left(\cos\left(\sqrt{\frac{b_2}{\sqrt{s}}}\,r\right)-\exp\left(-\sqrt{\frac{b_2}{\sqrt{s}}}\,r\right)\right)\\
&\quad+\left(a_2-\frac{280\,\pi\,\bar{\rho}}{9\,a_2\,l^2}\right)\exp\left(-\sqrt{\frac{a_2}{2\,\sqrt{s}}}\,r\right)\\
&\quad\left.\sin\left(\sqrt{\frac{a_2}{2\,\sqrt{s}}}\,r\right)\right],\\
(F_1-F_8)(r,s) &= \frac{9}{2\,\pi\,\Delta^{(1)}_{\rho\bar{\rho}}}\,\frac{1}{r\,s^{\frac{3}{2}}}\left[\frac{b_1}{2}\left(\cos\left(\sqrt{\frac{b_1}{\sqrt{s}}}\,r\right)\right.\right.\\
&\quad\left.-\exp\left(-\sqrt{\frac{b_1}{\sqrt{s}}}\,r\right)\right)\\
&\quad+a_1\,\exp\left(-\sqrt{\frac{a_1}{2\,\sqrt{s}}}\,r\right)\\
&\quad\left.\sin\left(\sqrt{\frac{a_1}{2\,\sqrt{s}}}\,r\right)\right],\\
(F_1+8\,F_8)(r,s) &= \frac{9}{4\,\pi\,\Delta^{(2)}_{\rho\bar{\rho}}}\,\frac{1}{r\,s^{\frac{3}{2}}}\left[\frac{b_2}{2}\left(\cos\left(\sqrt{\frac{b_2}{\sqrt{s}}}\,r\right)\right.\right.\\
&\quad\left.-\exp\left(-\sqrt{\frac{b_2}{\sqrt{s}}}\,r\right)\right)\\
&\quad+a_2\,\exp\left(-\sqrt{\frac{a_2}{2\,\sqrt{s}}}\,r\right)\\
&\quad\left.\times\sin\left(\sqrt{\frac{a_2}{2\,\sqrt{s}}}\,r\right)\right].
\end{aligned}
\tag{B.2}
$$

At this point, the inversion of the Laplace transform leads to the increasing oscillations which have also been shown in Sect. 4. In order to obtain a simpler answer, we take the space integral of the result. If we had a function of r damped at infinity we could simply take, as it has been noted, the limit $k^2 \to \infty$ in the Fourier transform, but we think that it is necessary to study in detail the case of the oscillating behavior. From (12) we see, decomposing the fractions, that the 'dangerous term' has the form

$$f(\mathbf{r}) = \int \mathrm{d}^3 k \, \frac{1}{ms} \, \frac{\gamma}{k^2\sqrt{s} - \lambda^2} \, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \tag{B.3}$$

with m, γ, λ positive constants. The Fourier transform is

$$\begin{aligned} f(\mathbf{r}) &= \frac{2\pi^2\gamma}{ms^{3/2}} \frac{1}{r} \cos(\lambda r s^{-1/4}) \\ &= \frac{2\pi^2\gamma}{ms^{3/2}} \frac{1}{r} \left[\sum_u \frac{1}{(4u)!} (\lambda r)^{4u} s^{-u} \right. \\ &\qquad \left. - \sum_v \frac{1}{(4v+2)!} (\lambda r)^{4u+2} s^{-v-1/2} \right]. \end{aligned} \tag{B.4}$$

At this point, we perform the inversion of the Laplace transform, so that we go from the variable s to the variable β and then integrate term by term in $\mathrm{d}^3 r$ with a damping factor $\mathrm{e}^{-\eta r}$. The final result is

$$\frac{32\pi^2\gamma\sqrt{\beta}}{m\eta^2} \left[\sum_u \frac{u+1/4}{\Gamma(u+3/2)} y^u - \sum_v \frac{u+3/4}{\Gamma(u+2)} y^{v+1/2} \right], \tag{B.5}$$

with $y = (\lambda/\eta)^4 \beta$. The series can be easily summed to give finally

$$\frac{32\pi^2\gamma\sqrt{\beta}}{m\eta^2} \left(\frac{1}{4} + y \frac{\partial}{\partial y} \right) \left[\mathrm{e}^y \frac{1}{\sqrt{y}} \mathrm{erf}(\sqrt{y}) - \frac{1}{\sqrt{y}} (\mathrm{e}^y - 1) \right]. \tag{B.6}$$

From the asymptotic behavior of the error function $1 - \mathrm{erf}\sqrt{x} \approx \frac{1}{\sqrt{\pi x}} \mathrm{e}^{-x}$ it follows that the limit $\eta \to 0$ is finite and thus the damping factor can be removed at the end of the calculations. Therefore, the simple procedure of working directly on the Fourier transforms is justified.

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